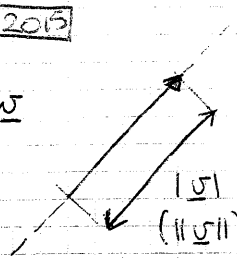
MECCANICA

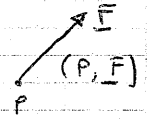
24-09-2015

velocity = vettore velocità
speed = modulo della velocità

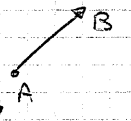


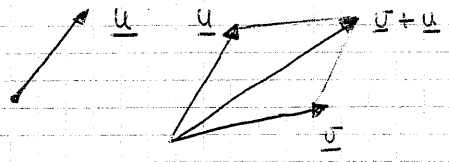
Modulo
direzione } in inglese riassunti
verso } entrambi con "direction"

Punto di applicazione (per vettori applicati e non liberi)

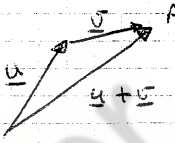


Differenza tra due punti è un vettore $B - A = \overrightarrow{AB}$





oppure



SOMMA VETTORIALE

somma testa coda

$\underline{v_1} + \underline{v_2} + \ldots = \underline{0} \iff$ punto di applicazione di $\underline{v_1}$ coincide con la testa di $\underline{v_n}$

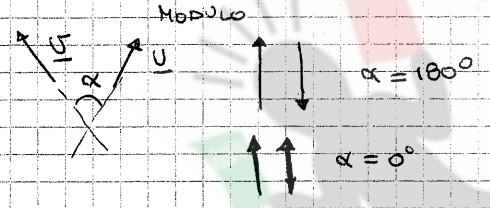


- $\underline{v} = a \cdot \underline{u} \quad a \in \mathbb{R}$
  $|\underline{v}| = |a| \cdot |\underline{u}|$

PRODOTTO PER SCALARE $\underline{F} = m\,\underline{a}$

RISULTATO: VETTORE

- $\underline{u} \cdot \underline{v} = |\underline{v}| \cdot |\underline{u}| \cdot \cos\alpha$



PRODOTTO SCALARE

RISULTATO: SCALARE

PRODOTTO VETTORIALE

- $\underline{u} \times \underline{v} = |\underline{v}| \cdot |\underline{u}| \cdot \sin\alpha = -\underline{v} \times \underline{u}$

MODULO

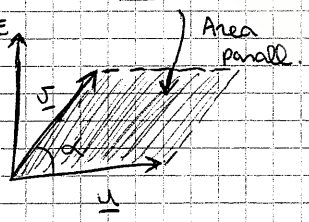
DIREZIONE: perpendicolare al piano individuato dai vettori

VERSO: entrante o uscente con la regola della mano destra

RISULTATO: VETTORE

Right Hand Rule

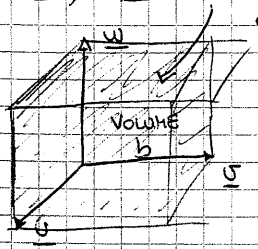
⊙ USCENTE ⊗ ENTRANTE



- $(\underline{u} \times \underline{v}) \cdot \underline{w} = b \in \mathbb{R}$

PRODOTTO TRIPLO

COMMUTATIVO



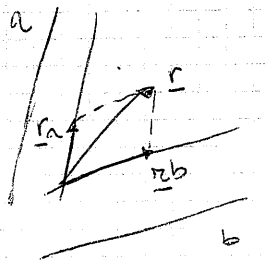
CIRCOLARITÀ

$$(\underline{u} \times \underline{v}) \cdot \underline{w} = \underline{w} \cdot (\underline{u} \times \underline{v}) = \underline{v} \cdot (\underline{w} \times \underline{u})$$

IDENTITÀ DI LAGRANGE

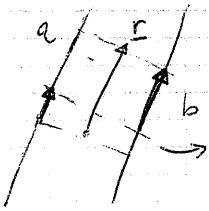
$$\underline{a} \times (\underline{b} \times \underline{c}) = \underline{b}(\underline{a} \cdot \underline{c}) - \underline{c}(\underline{a} \cdot \underline{b})$$

DECOMPOSIZIONE VETTORIALE



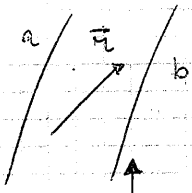
$\underline{r}_a + \underline{r}_b = \underline{r}$

scelta obbligata per la decomposizione in rette non //

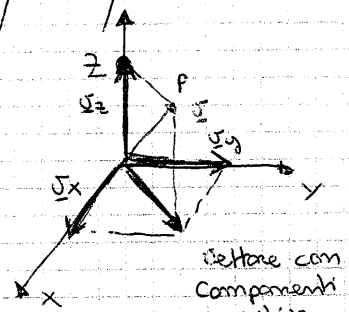


segno arbitrario se le due componenti sono //

$\underline{r}_a$ e $\underline{r}_b$ sono i VETTORI COMPONENTI



IMPOSSIBILE! Manca la componente ortogonale



vettore con le componenti positive. Le componenti sono le coordinate di P.

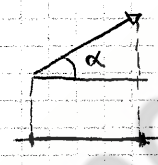
$\underline{v} = \underline{v}_x + \underline{v}_y + \underline{v}_z$ (VETTORI COMPONENTI)

LE COMPONENTI

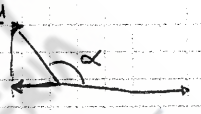
$v_x = \underline{v} \cdot \underline{i} \rightarrow \underline{v}_x = v_x \underline{i}$

$v_y = \underline{v} \cdot \underline{j} \rightarrow \underline{v}_y = v_y \cdot \underline{j}$

$v_z = \underline{v} \cdot \underline{K} \rightarrow \underline{v}_z = v_z \cdot \underline{K}$



$\underline{u} \cdot \underline{i} = |\underline{u}| \cos\alpha > 0$



$\underline{u} \cdot \underline{i} < 0$

$\underline{v} = (v_x, v_y, v_z) = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ $\quad |\underline{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$ $\quad \underline{v} = a\underline{v} = (a v_x, a v_y, a v_z)$

$|\underline{v}| = \sqrt{\underline{v} \cdot \underline{v}}$

$\underline{v} + \underline{s} = (v_x + s_x, v_y + s_y, v_z + s_z)$ $\quad \underline{v} \cdot \underline{s} = v_x s_x + v_y s_y + v_z s_z$

$[v_x \; v_y \; v_z] \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$

$\underline{v} \times \underline{s} = \det \begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_x & u_y & v_z \\ s_x & s_y & s_z \end{pmatrix} =$

$= \underline{i}(u_y s_z - v_z s_y) + \underline{j}(v_z s_x - v_x s_z) + \underline{k}(v_x s_y - v_y s_x)$

$L = \underline{F} \cdot \underline{s}$ $\quad \underline{u} \cdot (\underline{v} \times \underline{w}) = \det \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix}$

COSENI DIRETTORI

sia $\underline{\tau}$ un versore (al posto di $\underline{v}$ verde nel disegno sopra)

$\tau_x = \underline{\tau} \cdot \underline{i} \rightarrow \cos\alpha_x \cdot 1 \cdot 1$

$\tau_y = \underline{\tau} \cdot \underline{j} \rightarrow \cos\alpha_y \cdot 1 \cdot 1$

$\tau_z = \underline{\tau} \cdot \underline{k} \rightarrow \cos\alpha_z \cdot 1 \cdot 1$

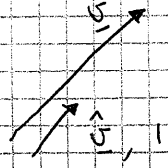
$\underline{\tau} = \tau_x \cdot \underline{i} + \tau_y \cdot \underline{j} + \tau_z \cdot \underline{k}$

$|\underline{\tau}| = 1 = \sqrt{\tau_x^2 + \tau_y^2 + \tau_z^2} = \sqrt{\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z}$

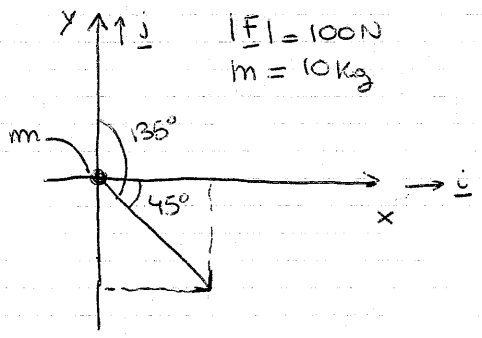
$1 = \cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z$

25-09-2015

Vettore a cui è associato un versore ($\hat{\underline{v}}$), (vers $\underline{v}$)



$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{1}{|\underline{v}|}(v_x, v_y, v_z)$



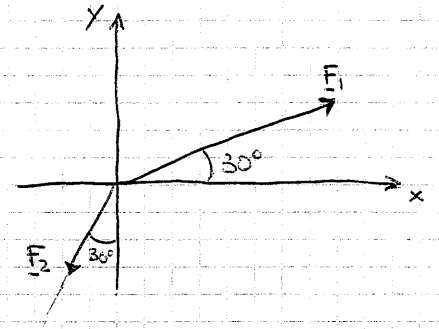
$|\underline{F}| = 100\,N$
$m = 10\,Kg$

$$\underline{F} = m\,\underline{a} \iff \begin{cases} F_x = \underline{F}\cdot\underline{i} = |\underline{F}|\cdot\cos\alpha = |\underline{F}|\cdot\frac{\sqrt{2}}{2} = 50\sqrt{2} \\ F_y = \underline{F}\cdot\underline{j} = |\underline{F}|\cdot\cos 135° = |\underline{F}|\left(-\frac{\sqrt{2}}{2}\right) = -50\sqrt{2}\,N \end{cases}$$

$$\underline{a} = \frac{1}{m}\cdot\underline{F} \iff \begin{cases} a_x = 5\sqrt{2}\ m/s^2 \\ a_y = -5\sqrt{2}\ m/s^2 \end{cases}$$

$$|\underline{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{25\cdot 2 + 25\cdot 2} = 10\ m/s^2$$

Si può anche scegliere un sistema di riferimento diverso.

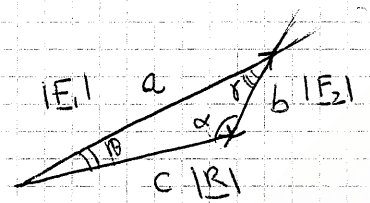


$|\underline{F}_1| = 20N$
$|\underline{F}_2| = 10N$

$$\underline{R} = \underline{F}_1 + \underline{F}_2 = (10\sqrt{3}-5,\ 10-5\sqrt{3})\,N = (12.3,\ 1.3)\,N$$

$$\underline{F}_1 = (10\sqrt{3},\ 10)\,N$$
$$\underline{F}_2 = (-5,\ -5\sqrt{3})\,N$$

$$|\underline{R}| = \sqrt{(10\sqrt{3}-5)^2 + (10-5\sqrt{3})^2} = 12.39\,N$$



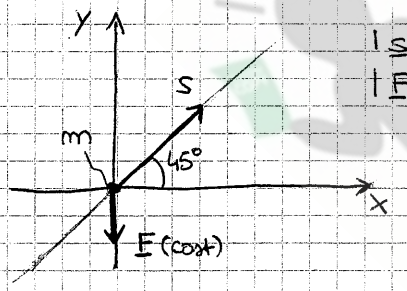
TEOREMA DEI SENI

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

TEOREMA DEI COSENI (DI CARNOT)

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$|\underline{R}|^2 = |\underline{F}_1|^2 + |\underline{F}_2|^2 - 2|\underline{F}_1||\underline{F}_2|\cdot\cos 30° \cong 12.39\,N$$

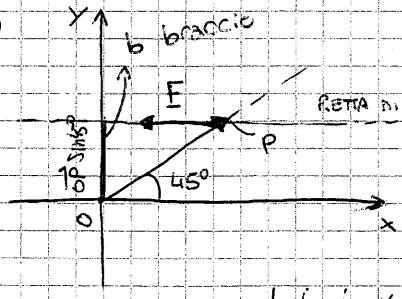


$|\underline{s}| = 0,5m$
$|\underline{F}| = 50N$

$\mathcal{L} = \underline{F}\cdot(\underline{d})\underline{s}$ omesso il d.s. perché sono su una traiettoria rettilinea.

$$\mathcal{L} = |\underline{F}|\cdot|\underline{s}|\cdot\cos 135° = -25\sqrt{2}\,J$$

$\underline{F} = (0,\ -50)\,N$
$\underline{s} = (\sqrt{2}/4,\ \sqrt{2}/4)\,m$ $\Rightarrow$ $\mathcal{L} = \underline{F}\cdot\underline{s} = -25\sqrt{2}/2\ Nm$



$(P, \underline{F})$: $\overline{OP} = 15m$, $|\underline{F}| = 10N$ $\quad \underline{M}_O = \overline{OP} \times \underline{F}$

$$\overline{OP} = \left(\overline{OP}\frac{\sqrt{2}}{2},\ \overline{OP}\frac{\sqrt{2}}{2}\right) = \left(\frac{15\sqrt{2}}{2},\ \frac{15\sqrt{2}}{2}\right)m$$

$$\underline{F} = (-10N,\ 0)\,N$$

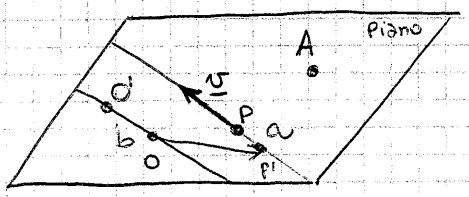
$$\underline{M}_O = \overline{OP}\times\underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 15\sqrt{2}/2 & 15\sqrt{2}/2 & 0 \\ -10 & 0 & 0 \end{vmatrix} = \underline{i}\cdot 0 + \underline{j}\cdot 0 + \underline{k}\cdot\frac{150\sqrt{2}}{2} = 75\sqrt{2}\ Nm\cdot\underline{k}$$

$$|\underline{M}_O| = 75\sqrt{2}\ Nm = |\underline{F}|\cdot|\overline{OP}|\cdot\sin 135° = 75\sqrt{2}$$

U V
ω W

01-10-2015

MOMENTI



b // a

$(P, \underline{v})$ a è retta di applicazione

Momento polare segue la legge di annullamento del prodotto vettoriale.

$\underline{M}_O = \overrightarrow{OP} \times \underline{v}$

Q è retta di applicazione se $\underline{v}$ è applicato a P'?

$\underline{M}'_O = \overrightarrow{OP'} \times \underline{v} = (\overrightarrow{OP} + \overrightarrow{OP'}) \times \underline{v} = \overrightarrow{OP} \times \underline{v} + \overrightarrow{PP'} \times \underline{v} =$

$= \underline{M}_O + \underline{0}$

- Principio di trasmissibilità di un vettore applicato

Comunque spostato il punto di applicazione di un vettore applicato lungo la stessa retta su cui giace, il momento rispetto ad un polo O giacente su b // a è lo stesso.

$\underline{M}_{O'} = \overrightarrow{O'P} \times \underline{v} = (\overrightarrow{O'O} + \overrightarrow{OP}) \times \underline{v} = \overrightarrow{O'O} \times \underline{v} + \overrightarrow{OP} \times \underline{v} = \underline{0} + \underline{M}_O$

- Anche traslando il polo parallelamente alla retta di applicazione il momento non varia.

$\underline{M}_A = \overrightarrow{AP} \times \underline{v} = (\overrightarrow{AO} + \overrightarrow{OP}) \times \underline{v} = \overrightarrow{AO} \times \underline{v} + \overrightarrow{OP} \times \underline{v} = M_O + AO \times \underline{v}$

$\underline{M}_A = \underline{M}_O + \overrightarrow{AO} \times \underline{v}$ Legge di trasporto del momento polare

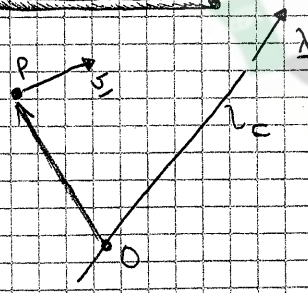
$\underline{M}_O$ può essere scritto analiticamente in forma matriciale

$|\underline{M}_O| = |\overrightarrow{OP}| \cdot |\underline{v}| \sin\alpha$

$|\overrightarrow{OP}| \sin\alpha$ = braccio della forza

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ OP_x & OP_y & OP_z \\ v_x & v_y & v_z \end{vmatrix} = \begin{matrix} \underline{i} & - & + \\ \underline{j} & - & + \\ \underline{k} & - & - \end{matrix}$$

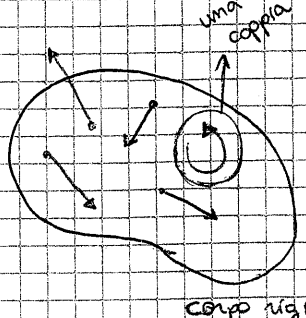
Momento assiale



$M_C = (\overrightarrow{OP} \times \underline{v}) \cdot \underline{\lambda}$ Prodotto triplo. Nullo se $\overrightarrow{OP}$, $\underline{v}$ e $\underline{\lambda}$ sono complanari.

Importante proprietà utili per descrivere fenomeni cinematici.

SISTEMI DI VETTORI APPLICATI



$S = (P_i, \underline{v}_i) \quad i = 1, \dots, n$

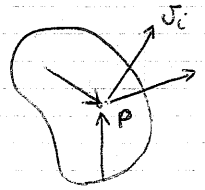
$$\underline{R} = \sum_{i=1}^{n} \underline{v}_i$$ RISULTANTE di un sistema di vettori applicati

$$\underline{M}_O = \sum_{i=1}^{n} \overrightarrow{OP_i} \times \underline{v}_i$$ MOMENTO POLARE risultante

Fondamentali per conoscere la dinamica (e la statica) di un qualsiasi corpo.

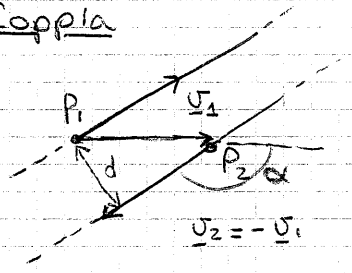
Teorema di Varignon

$$\underline{M_O} = \overrightarrow{OP} \times \sum_i \underline{v}_i = \overrightarrow{OP} \times R$$



Questo teorema vale se tutti i vettori applicati del sistema sono applicati su rette che si intersecano in un punto comune P.

Coppia



- rette di applicazione di $\underline{v}_1$ e $\underline{v}_2$ sono parallele
- $|\underline{v}_1| = |\underline{v}_2|$
- $\underline{R} = 0$
- $\underline{M_A} = \underline{M_O} + \cancel{\overrightarrow{AO} \times \underline{R}}$

Legge di trasporto del momento risultante M_A per una coppia di forze è indipendente dal polo scelto.

$$\underline{M_A} = \sum_i \overrightarrow{AP_i} \times \underline{v}_i = \sum_i (\overrightarrow{AO} + \overrightarrow{OP_i}) \times \underline{v}_i = \sum_i (\overrightarrow{AO} \times \underline{v}_i + \overrightarrow{OP_i} \times \underline{v}_i) = \sum_i \overrightarrow{AO} \times \underline{v}_i + \sum_i \overrightarrow{OP_i} \times \underline{v}_i =$$

$$= \overrightarrow{AO} \times \sum_i \underline{v}_i + \underline{M_O} = \underline{M_O} + \overrightarrow{AO} \times \underline{R} \quad \text{Legge di trasporto del momento risultante}$$

P_1: $\underline{M_{P_1}} = \overrightarrow{P_1P_2} \times \underline{v}_2 = |\overrightarrow{P_1P_2}||\underline{v}_2| \cdot \sin\alpha$

Con la regola della vite si comprende il funzionamento della coppia

Il modulo della coppia è dato da $|F_1| \cdot d$

La coppia è un vettore libero. Non applicato

Trinomio invariante τ

$\tau = \underline{M_O} \cdot \underline{R} = \underline{R} \cdot \underline{M_O}$

- trinomio: $R_x M_{O_x} + R_y M_{O_y} + R_z M_{O_z}$
- invariante: $\underline{R} \cdot \underline{M_A} = \underline{R} \cdot (\underline{M_O} + \overrightarrow{AO} \times \underline{R}) = \underline{R} \cdot \underline{M_O} + 0$

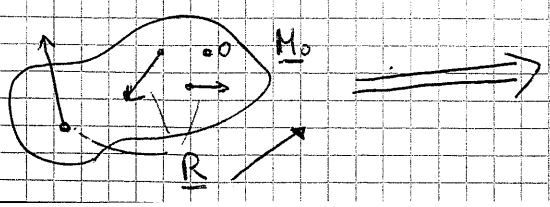
⇒ Due sistemi di vettori applicati si dicono equivalenti se hanno la stessa risultante e lo stesso momento applicato al polo O.

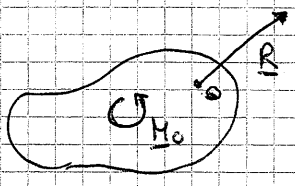
$\underline{R} = \underline{R}'$, $\underline{M_O} = \underline{M_O}'$ $\iff$ S_1 e S equivalenti

⇒ L'utilità pratica è quello di ridurre la complessità del sistema

⇒ Per passare da un sistema ad un altro:

1) Calcolo vettorialmente la risultante $\underline{R}$ in S;
2) Calcolo il momento rispetto ad un polo O in S;
3) Applico in S' una coppia che vale esattamente $\underline{M_O}$ e che non influisce nella risultante e non dipende dal polo O;
4) Sposto il vettore $\underline{R}$ in modo da applicarlo esattamente ad O (così non contribuisce al momento totale)





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